# Vector Field Notation

A vector **field** describes a vector value at **every** location in space. Therefore, we can denote a vector field as $\mathbf{A}(x,y,z)$, or $\mathbf{A}(\rho,\phi,z)$, or $\mathbf{A}(r,\theta,\phi)$, explicitly showing that vector quantity $\mathbf{A}$ is a **function** of position, as denoted by some set of coordinates.

However, as we have emphasized before, the **physical reality** that vector field $\mathbf{A}$ expresses is independent of the coordinates we use to express it. In other words, although the **math** may look **very different**, we find that:

$$\mathbf{A}(x,y,z) = \mathbf{A}(\rho,\phi,z) = \mathbf{A}(r,\theta,\phi).$$

Alternatively then, we typically express a vector field as simply:

$$\boxed{\mathbf{A}(\bar{r})}$$

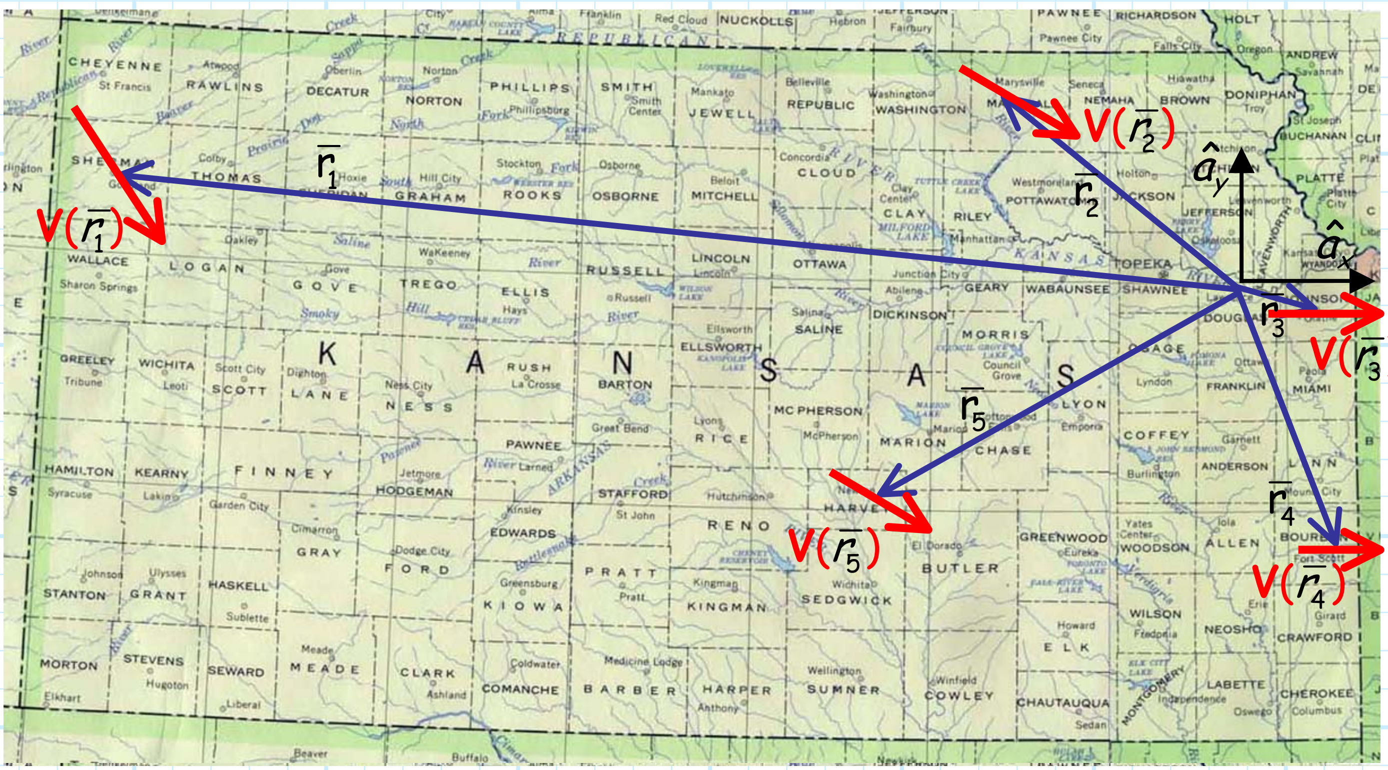
This **symbolically** says everything that we need to convey; vector $\mathbf{A}$ is a **function** of position—it is a **vector field**!

Note that the vector field notation $\mathbf{A}(\bar{r})$ does **not** explicitly specify a **coordinate system** for expressing $\mathbf{A}$. That's up to **you** to decide!

Now, in the vector field expression $\mathbf{A}(\bar{r})$ we note that there are two vectors: $\mathbf{A}$ and $\bar{r}$. It is **ridiculously important** that you understand what each of these two vectors represents!

> Position vector $\bar{r}$ denotes the location in space where vector $\mathbf{A}$ is defined.

For example, consider the vector field $\mathbf{V}(\bar{r})$, which describes the **wind velocity** across the state of Kansas.




In this map, the **origin** has been placed at Lawrence. The **locations** of Kansas towns can thus be identified using **position vectors** (units in miles):

$\bar{r}_1 = -400\,\hat{a}_x + 20\,\hat{a}_y$ → the **location** of Goodland, KS

$\bar{r}_2 = -90\,\hat{a}_x + 70\,\hat{a}_y$ → the **location** of Marysville, KS

$\bar{r}_3 = 30\,\hat{a}_x - 5\,\hat{a}_y$ → the **location** of Fort Scott,KS

$\bar{r}_4 = 40\,\hat{a}_x - 90\,\hat{a}_y$ → the **location** of Fort Scott,KS

$\bar{r}_5 = -130\,\hat{a}_x - 70\,\hat{a}_y$ → the **location** of Newton, KS

Evaluating the vector field $\mathbf{V}(\bar{r})$ at these locations provides the wind velocity **at** each Kansas town (units of mph).

$\mathbf{V}(\bar{r}_1) = 15\,\hat{a}_x - 17\,\hat{a}_y$ → the **wind velocity** in Goodland, KS

$\mathbf{V}(\bar{r}_2) = 15\,\hat{a}_x - 9\,\hat{a}_y$ → the **wind velocity** in Marysville, KS

$\mathbf{V}(\bar{r}_3) = 11\,\hat{a}_x$ → the **wind velocity** in Olathe, KS

$\mathbf{V}(\bar{r}_4) = 7\,\hat{a}_x$ → the **wind velocity** in Fort Scott, KS

$\mathbf{V}(\bar{r}_5) = 9\,\hat{a}_x - 4\,\hat{a}_y$ → the **wind velocity** in Newton, KS

Remember, from vector field $\mathbf{A}(\bar{r})$, we can the magnitude and direction of the discrete vector $\mathbf{A}$ that is **located** at the **point** defined by position vector $\bar{r}$.

This discrete vector $\mathbf{A}$ does **not** "extend" from the origin to the point described by position vector $\bar{r}$. Rather, the discrete vector $\mathbf{A}$ describes a quantity **at that point**, and that point only. The magnitude of vector $\mathbf{A}$ does **not** have units of distance! The **length** of the arrow that represents vector $\mathbf{A}$ is merely symbolic—its length has **no** direct physical meaning.

On the other hand, the position vector $\bar{r}$, being a directed distance, **does** extend from the origin to a specific **point** in space. The magnitude of a position vector $\bar{r}$ **is** distance—the length of the **position vector** arrow **has** a direct physical meaning!

Additionally, we should again note that a vector field need not be static. A **dynamic** vector field is likewise a function of **time**, and thus can be described with the notation:

$$\mathbf{A}(\bar{r},t)$$